BBN Overview

Statistics Review

$$n = \frac{g}{2\pi} \int_0^\infty \frac{p^2\,dp}{e^{(E-\mu)/T} \pm 1}$$

$+$: Fermions
$-$: Bosons

g: degeneracy
$E = \sqrt{m^2 + p^2}$
$\hbar = c = 1 = k_B$

Units: $\hbar = c = k_B = 1$

$$k_B = 8.617\ldots \times 10^{-5}\ \text{eV/K}$$
$$\approx 10^{-10}\ \text{MeV/K}$$
$$\hbar c = 1.97\ldots \times 10^{-11}\ \text{MeV}\cdot\text{cm}$$
$$M_{Pl} = \sqrt{\frac{\hbar c}{G}} \sim 10^{19}\ \text{GeV}$$

BBN: $T \sim$ 1 MeV $-$ 0.01 MeV
1s $-$ 1000s

Non-relativistic limit:

$$E \approx m + \frac{p^2}{2m}$$

$$n = g\left(\frac{mT}{2\pi}\right)^{3/2} e^{(\mu-m)/T}, \quad \rho = n\left(m + \tfrac{3}{2}T\right)$$

Ultra-Relativistic

$$E \approx p$$

$$n = f\,\frac{g\,\zeta(3)}{\pi^2}\,T^3$$

$$f = \begin{cases} 1 & \text{Bosons} \\ 3/4 & \text{Fermions} \end{cases}$$

$\zeta(3) \approx 1.202\ldots$

$$\rho = f'\,\frac{\pi^2}{30}\,g\,T^4$$

$$f' = \begin{cases} 1 & \text{Bos.} \\ 7/8 & \text{Fermion} \end{cases}$$

Radiation Domination $(T > eV)$

$$\rho \sim a^{-4}$$

only relativistic species

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho \quad , \quad \rho = g_*(T)\,\frac{\pi^2}{30}\,T^4$$

$$H(t) = \sqrt{\frac{8\pi^3 g_*}{90}}\,\frac{T^2}{M_{pl}}$$

$$g_*(T) = \sum_i^{\text{Bosons}} g_i\left(\frac{T_i}{T}\right)^4 + \frac{7}{8}\sum_i^{\text{Fermi}} g_i\left(\frac{T_i}{T}\right)^4$$

not all might be in Th. equill.

$$t \simeq 0.3\,s\,\frac{1}{\sqrt{g_*}}\left(\frac{MeV}{T}\right)^2$$

$g_* \approx 10.75$ for BBN

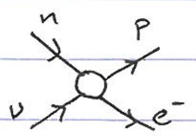
$e^{\pm}$, γ, $3(\bar{\nu}, \nu)$

BBN timeline

	10 MeV	– all in equillibrium (p, n)
$\sim 1\,s$	0.7 MeV	– $p-n$ freeze-out (n decay still)
		– D produced, but quickly killed by γ
$\sim 200\,s$	0.07	– D survives ("D Bottle neck")
		n decayed since freeze-out $\sim 10\%$
		– BBN starts
$\sim 10^3\,s$	0.03	– Nuclear rates drop below H : freeze out
		– primodial abundances fixed
		– ^4He $(Y_p) \approx 25\%$
		– D/H strongly depends on $\eta = \frac{n_b}{n_\gamma}$

BBN (1): 10 – 1 MeV – p/n freezeout

1) $n + \nu \leftrightarrow p + e^-$
2) $n + e^+ \leftrightarrow p + \bar{\nu}$
3) $n \leftrightarrow p + e^- + \bar{\nu}$ (decay)



Proton does not decay

1-3: Weak interaction keeps p+n in equillibrium, so long as

$$\Gamma \gtrsim H$$

$m_n \approx m_p \approx$ GeV : non-rel

$m_e \approx 0.5$ MeV : ultra-relativistic

$$\boxed{\frac{n_n}{n_p} = e^{-(m_n - m_p)/T} = e^{-\Delta/T}} \qquad (\Delta \simeq 1.3 \text{ MeV})$$

$T \gg 1$ MeV : equal numbers

- Around $T \lesssim$ MeV : p favoured

When leave equillibrium? $\Gamma \sim H$

$\Gamma = n\langle \sigma v \rangle$ ($n \sim T^3$ relativistic e^-)

$\sim G_F^2 T^5$

$$H \sim \sqrt{\frac{8\pi^3 g_*}{90}} \frac{T^2}{M_{pl}}$$

[Depends on # (light) neutrino species through g_*]

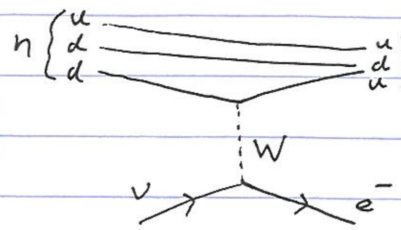
$T_{fo} \sim 0.7$ MeV (1 sec)

$\Rightarrow \dfrac{n_n}{n_p} \simeq e^{-1.3/0.7} = 0.16$

or : 1:6

But n keeps decaying:

Weak

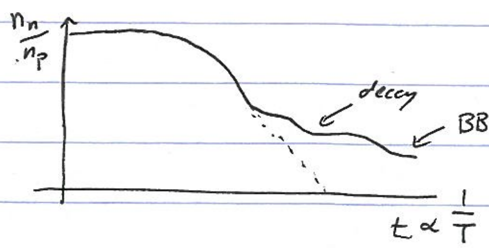


$m_W \sim 100$ GeV , $E \ll M_W$

$\sigma v \sim |A|^2 \sim g^4 \left(\frac{E}{M_W}\right)^2$

$= G_F^2 T^2$

$G_F \sim 10^{-5}$ GeV^{-2}



BBN (2): Deuterium 0.7 – 0.07 MeV

$D = {}^2H$
$T = {}^3H$

$$p + n \to D + \gamma$$

$$\left.\begin{aligned} p + D &\to {}^3He + \gamma \\ D + D &\to {}^3He + n \\ &\to T + p \\ {}^3He + D &\to {}^4He + p \\ T + D &\to {}^4He + n \end{aligned}\right\}$$

- Very fast strong interactions
- 4He tightly bound (Q_{4He} large)

⇒ All neutrons funnelled throug to 4He

- Depends on neutron number & D

[no stable $A = 5, 8$ ↳ Bridge]

$T + {}^4He \to {}^7Li$ (dominates for $\eta \lesssim 10^{-10}$)

${}^3He + {}^4He \to {}^7Be \to {}^7Li$ (Actually dominates)
(Slow) ↳ Coulomb barrier

$$p + n \leftrightarrow D + \gamma$$

$$n_i = g_i \left(\frac{mT}{2\pi}\right)^{3/2} e^{(\mu_i - m_i)/T} \qquad \text{for } p, n, D$$

Equillibrium: $\mu_D = \mu_n + \mu_p$ ($\mu_\gamma = 0$: massless boson)

$$g_D = 3, \quad g_p = g_n = 2$$
$$m_D \approx 2m_p \approx 2 \text{ GeV}$$

$$m_D = m_p + m_n - B, \qquad B = 2.22 \text{ MeV}$$

$$\frac{n_D}{n_n} \simeq n_p \, 6 \left(\frac{m_p T}{\pi}\right)^{-3/2} e^{B/T}$$

$$\eta = \frac{n_b}{n_\gamma} \sim 10^{-9} = 3 \times 10^{-8} \Omega_b h^2$$

Initially (~10 MeV) $n_p \approx n_n \approx 0.5 n_b$

low ~~High~~ T: D favored over neutrons... but

$$D + \gamma \leftrightarrow p + n$$

Deuterium "Bottleneck"

D destroyed by high-E photons

$$n_\gamma^* = n_\gamma(E \gtrsim B) \sim n_\gamma e^{-B/T}$$

Even though $T \lll B$

Significant # in high-E tail

D can't survive until

$$n_\gamma^* / n_D \lesssim 1$$

$$n_b = \eta \, n_\gamma \quad , \quad n_\gamma^* < n_b$$

$$e^{-B/T} < \eta$$

$$B/T > \ln(1/\eta)$$

$$\simeq \ln(10^9)$$

$T \lesssim 0.07$ MeV : Start BBN

Neutron Decay

p/n freeze-out: $T \sim 0.7$ MeV

between $0.7 \to 0.07$ MeV (~ 1s – 180s)

neutrons decay

$$\tau_n \approx 880\text{s}$$

$$\left(\frac{n}{p}\right)_{BBN} \approx \left(\frac{n}{p}\right)_{fo} e^{-t/\tau_n} = \frac{1}{6} e^{-180/880} \approx \frac{1}{7}$$

→ ^4He stable, tightly bound

→ efficiently produced (if D available)

→ ~All n end up in ^4He

$$Y_p = \frac{4 n_{He}}{n_b} \simeq 4\underbrace{\left(\frac{n_n/2}{n_p + n_n}\right)}_{\text{before BBN}} = \frac{2\ n/p}{1 + n/p} \qquad \frac{n}{p} \approx \frac{1}{7}$$

$\approx 25\%$

D $\sim 0.01\%$

Freeze-out ~ 0.03 MeV

→ details Complicated : depends strongly on η

Large η: - faster reactions
- more ^4He, less D

low η: - slower, more left-over D

$$D/H \propto \eta^{-1.6}$$

$$\simeq 2.6 \left(\frac{6}{\eta_{10}}\right)^{1.6}$$

obs: $2.5 \times 10^{-5} \Rightarrow \eta_{10} = 6$ $\boxed{\eta_{10} := 10^{10}\eta}$

Y_p, D/H : match observations very well

Li problem

$T + {}^4He \to {}^7Li$ Dominates for $\eta \lesssim 10^{-10}$

${}^3He + {}^4He \to {}^7Be \to {}^7Li$ Actual dominates 'now'

(also: decays)

Complicated dependance on η (and τ_n)

Predicted: $\frac{{}^7Li}{H} \sim 5 \times 10^{-10}$

observed $\sim 1.5 \times 10^{-10}$

3x discrepancy

- Complicated Pathways
- Mabye destroyed?