Weak Decoupling and Big Bang Nucleosynthesis : "the first 3 minutes"

Inflation

Baryogenisis

WIMP (?)

EW phase transition $\quad m_{W^\pm} \sim 100\ \text{GeV} \sim 10^{-11}\,s$

QCD " $\quad \sim 100\ \text{MeV} - 10^{-5}\,s$ (quarks bind into $p/n + \pi$)

Weak decoupling (neutrino) $\quad \sim \text{MeV} \quad \sim 1\,s \quad \Gamma_{weak} \lesssim H$

e^-/e^+ anhiclation $\quad \sim 0.5\ \text{MeV} \quad \sim 6\,s \quad$ photon heating

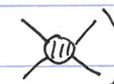
BBN $\quad \sim 0.1\ \text{MeV} \quad \sim 3\,min$

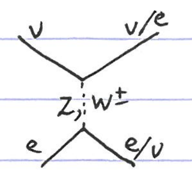
⋮

Recombination $\quad \sim 0.1\ \text{eV} \quad \sim 4\times10^5\ yrs \quad \Rightarrow$ CMB

Weak (+ neutrino) decoupling $\sim$ MeV

• Well after EW phase transition $\quad T \ll M_W$

$$\sigma v \sim \Big($$  $$\Big)^2 \sim G_F^2 T^2$$



$$\sim \frac{\sqrt{\alpha_W}\, E^2}{M_W^2}$$

$$G_F \sim \frac{\alpha_W}{M_W^2} \approx 1.17\times10^{-5}\ \text{GeV}^{-2}$$

Neutrinos ~~[illegible]~~ relativistic , only interact through weak:
$\nu_e + \bar{\nu}_e \leftrightarrow e^- + e^+$ and $e^- + \bar{\nu}_e \leftrightarrow e^- + \bar{\nu}_e$, $e^- + \nu \leftrightarrow e^- + \nu$

$$\rho_\nu = \frac{7}{8}\frac{\pi^2}{30}\, g\, T^4 \qquad g_* \approx 10.75$$

$g = 3\times2 \quad \nu, \bar{\nu}, (e,\mu,\tau) \qquad (e, e^+, \gamma, \nu)$

$$\boxed{H \sim 5\frac{T^2}{M_{Pl}}}$$

$$n_\nu = \frac{7}{8}\frac{1.2}{\pi^2}\, 6\, T^3$$

$$H = 1.66\sqrt{g_*}\,\frac{T^2}{M_{Pl}} \qquad R \approx n\langle\sigma v\rangle \sim H \quad (v\sim1) \qquad \boxed{n\langle\sigma v\rangle \sim G_F^2 T^5}$$

$$\Rightarrow T \sim \text{MeV}$$

$\Rightarrow$ Neutrinos + photons no longer in thermal equillibrium after $T_\gamma \sim$ MeV

- After this, no other rel. species interacts w/ ν
  (non-rel p/n too few to impact thermodynamics of ν)

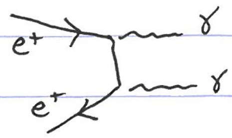
$\Rightarrow \quad n_\nu \sim a^{-3}$

- If nothing else changes T_γ, even though decoupled, would have $T_\gamma \simeq T_\nu$
  But

e/e$^+$ annihilation : 0.5 MeV

$T \lesssim 0.5$ MeV $\qquad\qquad m_e = 0.5$ MeV

$$e^- + e^+ \longrightarrow 2\gamma$$



energy injected into photons

From entropy conservation: $T_\nu \approx \left(\frac{4}{11}\right)^{1/3} T_\gamma$

$$g_* = 2 + \frac{7}{8} \times 2 N_{eff} \left(\frac{4}{11}\right)^{4/3} \approx 3.36$$

$N_{eff} = 3$ if instantly decoupled.
But some e^-/e^+ energy went to ν
$N_{eff} \approx 3.046$

Recombination

$$e^- + p \longleftrightarrow H + \gamma \quad : \boxed{\Delta E \approx 13\,\text{eV}}$$

$$T \sim \text{eV} \ll m_e, m_p, m_H$$

$$\Rightarrow \quad n_i^{eq} = g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} \exp\left([\mu_i - m_i]/T\right)$$

	m	g
e^-	0.5 MeV	2
p	GeV	2
H	GeV	4

$(\mu_\gamma = 0)$ Eliminate μ: $\mu_e + \mu_p = \mu_H$

$$\left(\frac{n_H}{n_e n_p}\right)_{eq} = \frac{g_H}{g_e g_p}\left(\left(\frac{m_H}{m_e m_p}\right)\frac{2\pi}{T}\right)^{3/2} e^{\Delta E/T}$$

$n_e = n_p$

$$\rightarrow \quad \boxed{\left(\frac{n_H}{n_p^2}\right)_{eq} = \left(\frac{2\pi}{m_e T}\right)^{3/2} e^{\Delta E/T}}$$

Define free fraction: $X = \dfrac{n_p^{free}}{n_b}$ $\quad \left(\begin{array}{l} n_p = n_e \\ n_b \equiv n_p + n_H \end{array}\right)$

$$n_b = \eta n_\gamma$$

Saha Equation:

$$\Rightarrow \quad \frac{1-X}{X^2} = \frac{2\zeta}{\pi^2}\eta\left(\frac{2\pi T}{m_e}\right)^{3/2} e^{\Delta E/T} \qquad \zeta \approx 1.202\ldots$$

Simple estimate

Naively: $T \sim 13\,\text{eV}$. but so many photons

$n_\gamma \gg n_p$ $\quad \hookrightarrow$ ~~tower~~ high-energy tail

Number of photons w/ $E > \Delta$ (fraction)

$$\frac{n_\gamma(E>\Delta)}{n_\gamma} = \frac{1}{2\zeta}\int_\Delta^\infty \frac{E^2\, dE}{e^{+E/T} - 1}\, dE$$

$$\approx 0.6 \left(\frac{\Delta}{T}\right)^2 e^{-\Delta/T}$$

Efficient Ionisation until $\quad n_b \approx n_\gamma(\varepsilon > \Delta)$

$$\Rightarrow \quad \frac{n_b}{n_\gamma} = \eta = \underbrace{0.6\left(\frac{\Delta}{T}\right)^2}_{\text{not important after log!}} e^{-\Delta/T} \quad \Rightarrow \quad T \approx \frac{-\Delta}{\ln(\eta)} = \frac{-13}{\ln(10^{-9})}\,\text{eV}$$

$$\approx 0.6\ \text{eV}$$