PHYS4080 — B.M. Roberts

Some select topics in particle astrophysics

- Production + interactions of particles in early universe
  - ↪ Impact on CMB (recombination), neutrinos
- Big Bang Nucleosynthesis
- Dark matter
  - production
  - evidence
  - direct + Indirect detection
- Possibly: baryogenisis, inflation

- Very wide field ↗ just a subset.
- Touch on these, more detail for some.

- Background: particle physis + statistical mechanics
- Equillibrium → out-of-equillibrium: freeze-out + decoupling
- WIMP miracle: production of 'Thermal Relic' dark Matter
- Big bang nucleosynthesis + recombination
- Indirect detection of dark matter
- Direct detection of dark matter

Particle - Physics 101

Standard Model

Fermions (spin $1/2$) : Leptons $e^-, e^+, \nu, .. \tau, \mu^-$ MeV ~ GeV

quarks $u, d ...$ ~ MeV - 100 GeV

(~~Hadrons~~ Baryons:* p, n) ~ GeV

(Hadrons: * Not always spin $1/2$ (or even Fermions!))

Bosons spin 1 (or 0)

γ — EM

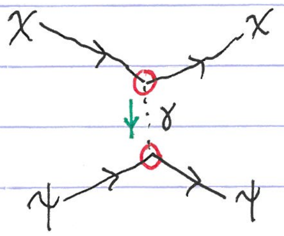
$Z, W^\pm$ ~100 GeV — Weak

g — Strong

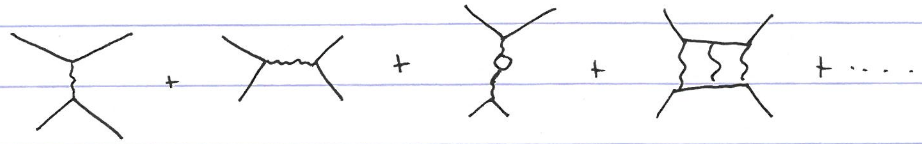
H — Higgs

Note: Missing:
- DM
- DE
- Source of baryon assymetry!

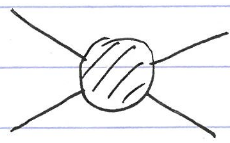
Interactions between Fermions (+ hadrons ...) mediated by bosons



- Feynman diagram represents an Amplitue $P \sim |A|^2$
- Vertex + propagator ... Very complicated
- generally, must consider all: with same initial/final



any number of in/out particles "usually" $2 \to 2$ or $1 \to 2$ (for us)



"~~effective~~ effective" vertex

→ either ~~taken~~ from theory

→ treat as observable / parameter

Scattering

$p^\mu_i = p^\mu_f \Rightarrow E_i = E_f$ and $\vec{p}_i = \vec{p}_f$ *

Elastic (initial particles = final)

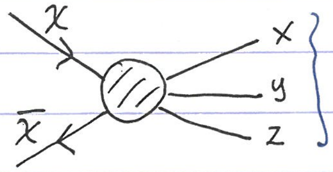
Vs inelastic (chang in particle number/species)

* also: charge, spin etc. conserved

Electric, baryon #, Lepton #

Two important cases

1. Anhialation (/production) : particle p + anti-particle $\bar{p}$



Any number

time ⟶

Relativistic : either way $\qquad E_i = E_f \qquad E = \sqrt{p^2c^2 + m^2c^4}$

Non-relativistic : constrained $\qquad \sum_f m_f \leq 2m_\chi$

$E \approx m$

Scattering cross-section (any scattering, not just annihilation) $\quad 2 \to N$

$$A + B \to x, y, z \ldots$$

R

Rate, (per A particle)

number density (#/V)

relative velocity

$$R = \underbrace{n_B \cdot v_{rel}}_{\text{\# of B's passing through area, each sec}} \times \boxed{\sigma}$$

σ: cross section

↳ contains all particle physics

$$\frac{1}{T} = \frac{1}{L^3}$$

Total rate: $\qquad R_T = n_B \cdot v \cdot \sigma \cdot N_A$

$\boxed{\langle \sigma v \rangle : \text{thermally averaged } \sigma}$

Rate, per unit volume:

$$\boxed{\mathcal{R} = n_A n_B \langle \sigma v \rangle}$$

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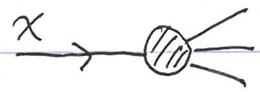
2. Decay $\quad 1 \to N$

always: $\sum_f m_f < m_\chi$



$R = n_\chi \Gamma \qquad$ (defines Γ)

↑ per unit volume

Units

$$c = \hbar = k_B = 1$$
$$G = \frac{1}{M_{P\ell}^2}$$

- Measure things in energy, GeV
- $[Q] = \text{GeV}^n$

  ↳ "Mass dimension" n

$c = 1$: $[\text{Length}] = [\text{Time}]$

$[\text{Mass}] = [\text{Energy}]$

$\hbar = 1$: $[\text{Energy}] = \frac{1}{[\text{Length}]}$

$k_B = 1$: $[\text{Energy}] = [\text{Temperature}]$

Convert: combos of $\hbar, c, k_B, M_{P\ell}$:

$$\boxed{\begin{aligned} \hbar c &\approx 197 \ \text{MeV} \cdot \text{fm} \\ &\approx 2 \times 10^{-14} \ \text{GeV} \cdot \text{cm} \\ M_{P\ell} \equiv \sqrt{\frac{\hbar c}{G}} &\approx 10^{19} \ \text{GeV} \\ k_B &\approx 8.6 \times 10^{-5} \ \text{eV/K} \\ &\approx 10^{-10} \ \text{MeV/K} \end{aligned}}$$

$\hbar = c = 1$

$\text{GeV} \approx 5.07 \times 10^{13} \ \text{cm}^{-1} \ (\hbar c)$

↑ usefull for project 4!

$c = 3 \times 10^{10} \ \text{cm/s}$

σv example $\quad L^2 \times \frac{L}{T} \longleftrightarrow \frac{1}{\text{GeV}^2}$: Mass dimension: -2

From $\text{GeV} \longmapsto \text{cm}^3/\text{s}$

$$\sigma v \xrightarrow[\text{dimensionless}]{\text{(A) make}} \frac{\sigma (v/c)}{(\hbar c)^2 \text{GeV}^{-2}}$$

(B) then "multiply by 1":

$$= \frac{\sigma (v/c)}{(\hbar c)^2 \text{GeV}^{-2}} \times \frac{c}{3 \times 10^{10} \text{cm/s}} \frac{(5.07 \times 10^{13} \ \text{cm}^{-1})^2 (\hbar c)^2}{\text{GeV}^2}$$

$$= \frac{\sigma v}{\text{cm}^3/\text{s}} \ 8.57 \times 10^{16}$$

$$\Rightarrow \quad \frac{\sigma v}{\text{cm}^3/\text{s}} = \left(\frac{\sigma v}{\text{GeV}^{-2}} \right) \times 1.17 \times 10^{-17}$$

Cosmological Thermodynamics

- Scattering + decays : allow energy exchange + particles creation

elastic : Kinetic equill. } Thermal eq.
inelastic : Chemical + Kinetic

Can equillibrate if $\boxed{R > H}$

$H = H(t)$, $R = R(n(t), T(t))$

Equillibrium

Distribution Function

$$n = \frac{g}{(2\pi)^3}\int \underbrace{f(p)}\, d^3p$$

$$= \frac{g}{2\pi^2}\int_0^\infty f(p)\, p^2\, dp$$

$$\rho = \frac{g}{2\pi^2}\int f(p)E(p)\, p^2\, dp$$

$$E(p) = \sqrt{p^2 + m^2}$$

$$f(p) = \frac{1}{e^{(E-\mu)/T} \pm 1}$$

g: degeneracy (# spin)

$+1$: Fermions

-1 : Bosons

μ: Chemical Potential

~ 'Cost' of changing # particles

$\mu \equiv \frac{\partial E}{\partial N}$, rarely need

Equillibrium: $A,B \Leftrightarrow C,D$

$\mu_A + \mu_B = \mu_C + \mu_D$

+ eliminate

$\mu_\gamma \simeq 0$, $\mu_\nu \approx 0$

Ultra relativistic: $\mu \ll E$

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(1) Non-rel limit, $T \ll m$, $E \simeq m + \frac{3}{2}T$

$$n = g\left(\frac{mT}{2\pi}\right)^{3/2} e^{-(m-\mu)/T}$$

$\sim e^{-m/T}$: Suppression!

$$\rho = nE$$

(2) Ultra-relativistic : $T \gg m$, $E \simeq p \sim T$ — Dominate (relativistic species)

$$n = f_n \frac{\zeta}{\pi^2} g T^3$$

$$\rho = f_\rho \frac{\pi^2}{30} g T^4$$

$f_n = \begin{cases} 1 & \text{bosons} \\ 3/4 & \text{Fermions} \end{cases}$, $\zeta \simeq 1.202\ldots$

$f_\rho = \begin{cases} 1 & \\ 7/8 & \text{Fermions} \end{cases}$

Radiation domination : $T \gtrsim eV$

$$\rho_m \sim a^{-3}$$
$$\rho_r \sim a^{-4}$$

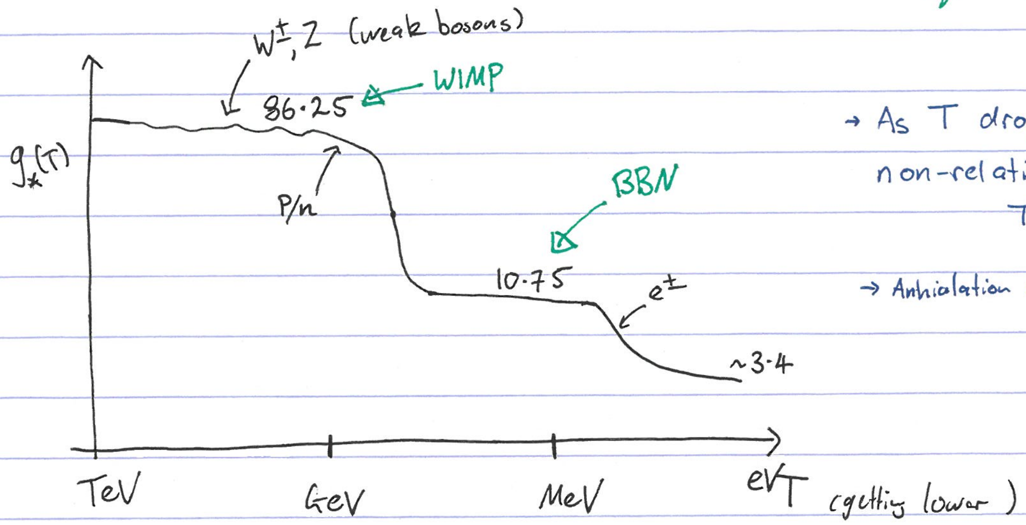
Flat: $$\left(\frac{\dot a}{a}\right)^2 = H^2 = \frac{8\pi G}{3}\rho = \frac{8\pi}{3 M_{Pl}^2}\rho$$

effective # of relativistic degrees of freedom

Radiation $\equiv$ Relativistic; $$\rho = g_* \frac{\pi^2}{30} T^4$$

$$g_*(T) \simeq g_{eff}^{rel} = \sum_b^{\text{Bosons}} g_b \underbrace{\left(\frac{T_b}{T}\right)^4} + \frac{7}{8}\sum_f^{\text{Fermions}} g_f \underbrace{\left(\frac{T_f}{T}\right)^4}$$

→ May not all be in thermal equillibrium



→ As T drops, particles become non-relativistic when $T \sim m$

→ Anhialation continues, $n \sim e^{-m/T}$

Freidman: $$H(T) = \sqrt{\frac{8\pi^3}{90} g_*(T)}\,\frac{T^2}{M_{Pl}} \approx 1.66\sqrt{g_*}\,\frac{T^2}{M_{Pl}}$$

$$\left(\rho_{crit} \simeq \frac{3H_0^2}{8\pi G} \sim 10^{-5} h^2 \frac{GeV}{cm^3}\right) \approx 8 \times 10^{-47} h^2\, GeV^4$$

$$t \approx 0.3s \frac{1}{\sqrt{g_*}}\left(\frac{MeV}{T}\right)^2$$ during Rad. dom.

(s: entropy density
h: entropic d.o.f, $h \approx g$)

$$s = \sum_i \frac{\rho_i + P_i}{T_i} = \frac{2\pi^2}{45} h_*(T)\, T^3$$

Freeze-out + Decoupling

In equillibrium non-rel species exponentially suppressed : $n \sim e^{-m/T}$ $\rho \sim n$

rel species not $n \sim T^3$, $\rho \sim T^4$

$T_{today} \sim 3\,K \sim 10^{-4}\,eV$ .... all non-rel, except γ

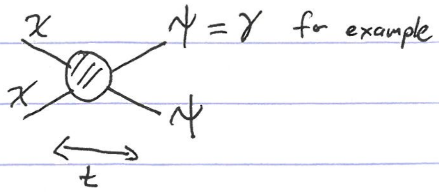
So, why not just photons?

Can't remain in equillibrium if $R < H$: "decouple" from thermal bath

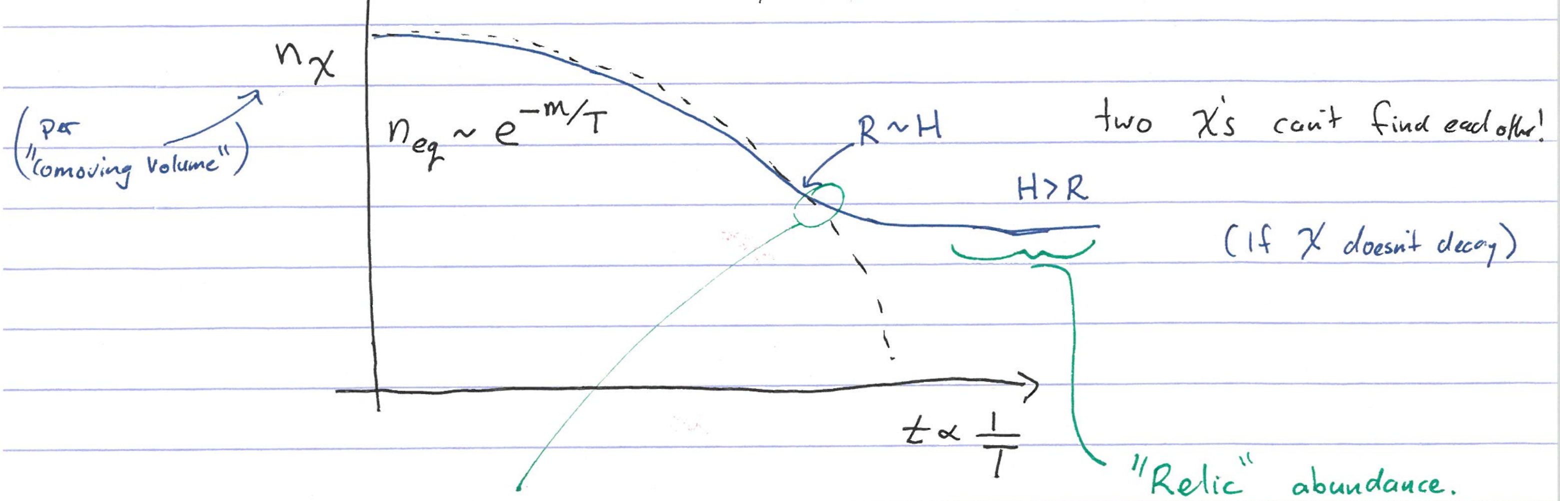
per particle $= n\langle\sigma v\rangle$

Consider χ, ψ $m_\chi \gg m_\psi$

$$\chi + \bar{\chi} \rightleftharpoons \psi + \bar{\psi}$$



$T < m_\chi$



Freeze out!

$\chi + \bar{\chi}$ can no loger occur if $H \gg R$

nb: $R \propto \sigma \cdot n$

Larger σ ⇒ later freezeout ⇒ fewer particles remain!

↳ Crucial for DM!

→ BBN − production of p/n + light elements] Next Lectures

→ recombination $p + e^- \to H + \gamma$ + CMB

→ WIMP 'miracle' : DM production | Project 4

Boltzmann Eq & DM thermal production

Consider No interactions, $N =$ particles per comoving volume

$N = a^3 n$ const

$$\frac{dN}{dt} = 0 \Rightarrow \frac{d(a^3 n)}{dt} = 3a^2 \dot{a} n + a^3 \dot{n} = 0$$

$$= 3\frac{\dot{a}}{a} n + \dot{n} = 0$$

$$\frac{dn}{dt} \cancel{n} + 3H = 0$$

"collision" term

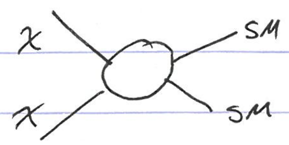
Add interactions $$\boxed{\frac{dn_i}{dt} + 3H = C_i(\{n_j\})}$$ Boltzmann Equation

$$C = R_{Production} - R_{anhialation} - R_{decay}$$

Dark Matter: WIMPs

$$\chi + \bar{\chi} \longleftrightarrow \psi + \bar{\psi}$$

$$m_\chi > m_\psi$$



No decay (DM must be stable)

$$R_{\cancel{prod}\,ann} = \langle \sigma v \rangle n_\chi^2$$

$$R_{prod} = \langle \sigma v \rangle n_{??}^2$$

In equillibrium $\frac{dN}{dt} = 0$ $\quad n_? = n_{eq}$

$$\frac{dn}{dt} = -3Hn + \langle \sigma v \rangle (n_{eq}^2 - n^2)$$

Aside: Practical calculation details

In practice easier to work in dimensionless + comoving units

(details not important):

$$s = \frac{S}{a^3}, \qquad Y = \frac{n}{s}, \qquad x = {}^{m}\!/_{T} \propto t$$

comoving entropy density comoving abundance

$$s = \frac{\rho + P}{T} = \frac{2\pi^2}{45} g_* T^3$$

effective rel. d.o.f

$$H \approx 1.66 \sqrt{g_*} \frac{T^2}{M_{Pl}} \qquad (g_* \approx 86.25)$$

$$\Rightarrow \boxed{\frac{dY}{dt} = \left(\sqrt{\frac{\pi}{45}} M_{Pl} \sqrt{g_*}\right) \frac{m}{x^2} \langle \sigma v \rangle \left(Y_{eq}^2(x) - Y^2(x) \right)}$$

Simple case: $$Y_{eq}(x) \approx \frac{45}{4\pi^4} \frac{x^2}{g_*} (2J_\chi + 1) K_2(x)$$

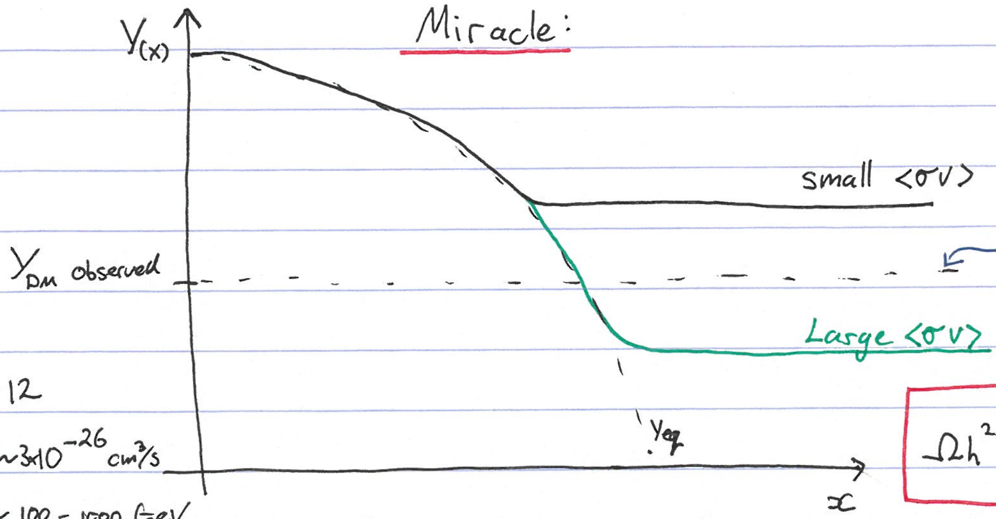
(Modified Bessel fn of 2nd Kind)

$J_\chi = 1/2 =$ spin

Convert $Y \to \Omega h^2$ for late times:

$$Y_{(now)} \approx 3.63 \times 10^{-9} \left[\frac{GeV}{m_\chi}\right] \Omega h^2$$

Miracle:



Weak-Scale:

$m_\chi \sim 100 - 1000$ GeV

$\langle \sigma v \rangle \approx 3 \times 10^{-26}\, cm^3/s$

Matches observed!!

$\sim \langle \sigma v \rangle \sim 3 \times 10^{-26}\, cm^3/s$

$\Omega h^2 \approx 0.12$

$\Rightarrow \langle \sigma v \rangle \sim 3 \times 10^{-26}\, cm^3/s$

$m_\chi \sim 100 - 1000$ GeV

$$\boxed{\Omega h^2 \propto \frac{1}{\langle \sigma v \rangle}} \sim \frac{m_\chi^2}{g^4}$$

$(g \sim 0.6)$